

Spin-Polarization Response Functions in High-Energy ($\vec{e}, e' \vec{p}$) Reactions

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Abstract

Spin-polarization response functions are examined for high-energy ($\vec{e}, e' \vec{p}$) reaction by computing the full 18 response functions for the proton kinetic energy $T_{p'} = 0.515$ GeV and 3.179 GeV with an ^{16}O target. The Dirac eikonal formalism is applied to account for the final-state interactions. The formalism is found to yield the response functions in good agreement with those calculated by the partial-wave expansion method at 0.515 GeV. We identify the response functions that depend on the spin-orbital potential in the final-state interactions, but not on the central potential. Dependence on the Dirac- or Pauli-type current of the nucleon is investigated in the helicity-dependent response functions, and the normal-component polarization of the knocked-out proton, P_n , is computed.

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I. INTRODUCTION

In a hard $(e, e'p)$ reaction, involving a few GeV/c or larger momentum transfer, the knocked-out proton experiences a strong, final-state interaction because of the large pN cross sections (30-45 mb) corresponding to a short mean-free path of about 1.5 fm. Perturbative quantum chromodynamics suggests, however, the possibility of color transparency^{1,2}, in which the knocked-out proton undergoes little final-state interaction in the hard $(e, e'p)$. The knocked-out proton would be small (about the inverse of the momentum transfer) and color singlet, and would interact weakly with the other nucleons in the nucleus through the color Van der Waals mechanism. This possibility has received much attention theoretically³⁻⁷ and experimentally⁸⁻¹¹.

Response functions from the $(e, e'p)$ reaction are affected greatly by the final-state interaction of the knock-out proton. Once the initial nuclear wave function is known (or assumed to be known), the response functions provide information of the final-state interaction, or the propagation of the knocked-out proton in nuclei. Polarization measurements in the $(\vec{e}, e'p)$ and $(\vec{e}, e'\vec{p})$ can provide detailed information on the process through the polarization response functions. The polarization measurements in the GeV region are thus of great interest, and are planned to be carried out at the Thomas Jefferson National Accelerator Facility (TJNAF)^{12,13}.

Theoretical investigation of the polarization response functions has been focused on the low proton energy of several hundred MeV or less¹⁴⁻¹⁶. In the GeV region, though a few calculations have been carried out for the response functions of $(\vec{e}, e'p)$ in the last few years¹⁸⁻²⁰, no calculation is yet available for $(\vec{e}, e'\vec{p})$.

In this paper we report the first calculation of the full set of the eighteen spin response functions for $(\vec{e}, e'\vec{p})$ in the GeV region, by incorporating spin-dependent, final-state interactions. We do not address the issue of the color transparency, but we calculate the response functions for the proton from different nuclear shell orbits and investigate their dependence on the spin-orbit interaction and the proton form factors. We also discuss briefly

the response functions of $(\vec{e}, e' \vec{n})$.

As in the works in low energies^{14–16}, we employ the Dirac formulation for the bound-state wave functions and as in the previous works in the GeV region^{18–20}, we apply the Dirac eikonal formalism to the knocked-out proton wave function in the final state. As in previous works, we neglect some physically important effects such as those that are due to off-shell effects and the current conservation. We do it in this exploratory work so as to establish bench-mark results, which could be compared with more refined calculations in future.

Though these effects are expected to be by no means negligible in the GeV region, the physics tends to become considerably simpler in comparison to that in the low-energy region: The Dirac eikonal formalism has been successfully applied to the spin-asymmetry analysis of the proton-nucleus elastic scattering for the proton energy of 0.515 GeV¹⁷. In the last few years, the same formalism has been applied to the $(\vec{e}, e' p)$ reaction in the GeV region^{18–20}. Later in this work, we explicitly demonstrate that the eikonal formalism is valid in the calculation of the response functions by comparing with the partial-wave decomposition method at 0.515 GeV. Our demonstration disagrees with the result of Ref.²¹, but agrees with its more recent result²².

Being consistent with the Dirac eikonal description of the knocked-out proton, we use the Hartree mean-field wave function of the Walecka model²³ for the bound-state proton. We thus neglect the nuclear correlation throughout this work. Though the significance of the correlation effects on the high-energy $(e, e' p)$ reactions is in debate^{24,25}, the effects appear to be small, once the other effects such as the finite range of the proton-nucleon interactions is included²⁵.

In Section II we review briefly the formalism for the $(\vec{e}, e' \vec{p})$ reaction and the Dirac eikonal method. In Section III, the numerical results of the 18 spin-dependent response functions are presented, together with the examination of the role of the spin-orbit force and the dependence on the structure of the electromagnetic current operator. The summary and the conclusion are given in Section IV.

II. QUASI-ELASTIC ELECTRON SCATTERING FORMALISM

A. Spin-Dependent Response Functions

In this work, we follow the convention and notations of the $(\vec{e}, e'\vec{p})$ kinematics, which were used by Picklesimer and Van Orden¹⁵. For convenience, the various kinematical quantities are illustrated in Fig. 1. The definition of the quantities are as follows: The four momenta of the incoming and the outgoing electron are denoted as k and k' , respectively; the photon momentum is $q = k - k'$ with $q^2 \equiv q_0^2 - \mathbf{q}^2 < 0$ (space-like); and the four momentum of the knocked-out proton is p' . We also denote e , m_e , and M to be the electron charge, the electron mass, and the nucleon mass, respectively, and $E_{\mathbf{p}'} = (\mathbf{p}'^2 + M^2)^{1/2}$ to be the on-shell energy of the proton. We follow the Bjorken-Drell convention²⁷ of gamma matrices and Dirac spinors, in which the normalization condition is $\bar{u}(k, s)u(k, s) = 1$ for the Dirac plane waves.

In the following, we sketch the formalism on which our calculation is based. The formalism is of the standard, as described in Ref. (15), but since it is rather involved, we wish to present it here for the sake of specifying notations and of clarifying the approximations involved in the quantities we calculate.

We assume 1) that the interaction between a proton in the nucleus and the electron is the one-photon exchange, and 2) that the nuclear current consists of one-body currents. We can then write the $(\vec{e}, e'\vec{p})$ cross section for h and $\hat{\mathbf{s}}$, the initial electron helicity and the spin polarization of the knocked-out proton, respectively, as

$$\left(\frac{d^3\sigma}{dE_{\mathbf{k}'}d\Omega_{\mathbf{k}'}d\Omega_{\mathbf{p}'}} \right)_{h, \hat{\mathbf{s}}} = \frac{M|\mathbf{p}'|}{(2\pi)^3} \left(\frac{d\sigma}{d\Omega_{\mathbf{k}'}} \right)_{Mott} \sum_a \int dE_{\mathbf{p}'} |\mathcal{M}_a|^2 \delta(E_{\mathbf{p}'} - q^0 - M + \varepsilon_a), \quad (1)$$

summing over the occupied nuclear shell-orbits (a 's) in the single-particle description of the nucleus. (ε_a is the binding energy in the a shell.) Here, the Mott cross section is

$$\left(\frac{d\sigma}{d\Omega_{\mathbf{k}'}} \right)_{Mott} = \left(\frac{e^2 \cos \frac{\theta}{2}}{8\pi |\mathbf{k}| \sin^2 \frac{\theta}{2}} \right)^2, \quad (2)$$

where θ is the electron scattering angle. The square of the transition amplitude for the knock-out proton in the a -shell, $|\mathcal{M}_a|^2$, is written as a product of the leptonic and nuclear tensors:

$$|\mathcal{M}_a|^2 = \eta_{\mu\nu} W_a^{\mu\nu}. \quad (3)$$

The leptonic tensor is defined by

$$\begin{aligned} \eta_{\mu\nu} &= m^2 \sum_{s'_e} [\bar{u}(k, s_e) \gamma_\mu u(k', s'_e)] [\bar{u}(k', s'_e) \gamma_\nu u(k, s_e)] \\ &= \frac{1}{2} (k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' - i h \epsilon^{\mu\nu\lambda\rho} k'_\lambda k_\rho), \end{aligned} \quad (4)$$

where s_e and s'_e are the initial and final spins of the electron, respectively, and $\epsilon^{\mu\nu\lambda\rho}$ is an antisymmetric, fourth-rank tensor. Note that the electron mass is neglected in the second step of Eq. (4).

The nuclear tensor $W_a^{\mu\nu} \equiv W_a^{\mu\nu}(q; \mathbf{p}', \hat{\mathbf{s}})$ depends on q , \mathbf{p}' , and $\hat{\mathbf{s}}$, as well as on the quantum number of the a -shell orbit, and is written in terms of the matrix element of the nuclear current operator J^μ ,

$$W_a^{\mu\nu}(q; \mathbf{p}', \hat{\mathbf{s}}) = \sum_{j_z} J_{a', \hat{\mathbf{s}}}^{\mu\dagger}(q, \mathbf{p}') J_{a', \hat{\mathbf{s}}}^\nu(q, \mathbf{p}'), \quad (5)$$

where a' is the quantum number of the proton (that is to be knocked out) in the a -shell, including j_z , the z -component of its total angular momentum. The matrix element of J_μ is given by

$$J_{a', \hat{\mathbf{s}}}^\nu(q, \mathbf{p}') = \langle \psi_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)} \Psi_F(A-1, a') | j^\nu(q) | \Psi_I(A) \rangle. \quad (6)$$

Here, $\psi_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)}$ is the scattered wave function of the knocked-out proton that satisfies the incoming boundary condition. $\Psi_I(A)$ is the initial, ground-state nuclear wave function, and $\Psi_F(A-1, a')$ is the final-state nuclear wave function with one hole that carries the quantum number a' . $j^\nu(q)$ is the one-body current operator to be specified shortly.

We introduce a Möller-type operator, $\Omega^{(-)}$, that converts the Dirac plane wave to the distorted wave with the incoming boundary condition,

$$\psi_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)} = \Omega^{(-)} u_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)}. \quad (7)$$

Note that $\Omega^{(-)}$ is not unitary, as seen explicitly in Subsection II B. Equation (7) now allows us to write the nuclear tensor as the diagonal element of the Dirac plane-wave spinor basis, $|u_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)}\rangle$:

$$W_a^{\mu\nu}(q, \mathbf{p}', \hat{\mathbf{s}}) = \text{Tr}[P_{\hat{\mathbf{s}}}(\mathbf{p}') \cdot \omega_a^{\mu\nu}(q)]. \quad (8)$$

Here, the spin-projection operator $P_{\hat{\mathbf{s}}}(\mathbf{p}')$ is defined in terms of the Dirac plane-wave spinors as

$$P_{\hat{\mathbf{s}}}(\mathbf{p}') = |u_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)}\rangle \langle \bar{u}_{\mathbf{p}', \hat{\mathbf{s}}}^{(-)}| \quad (9)$$

$$= \left(\frac{\not{\mathbf{p}}' + M}{4M} \right) (1 + \gamma^5 \not{\hat{\mathbf{s}}}), \quad (10)$$

where the space-like, spin four vector s^μ is orthogonal to the momentum four vector of the knocked-out proton and is normalized to unity. s^μ is related to the spin vector in the rest frame of the proton, $\hat{\mathbf{s}}$, as

$$s = \left(\frac{\hat{\mathbf{s}} \cdot \mathbf{p}'}{M}, \hat{\mathbf{s}} + \frac{\hat{\mathbf{s}} \cdot \mathbf{p}'}{M(E_{\mathbf{p}'} + M)} \mathbf{p}' \right). \quad (11)$$

$\omega_a^{\mu\nu}(q)$ is the nuclear tensor in the Dirac plane-wave spinor space,

$$\omega_a^{\mu\nu}(q) = \sum_{j_z} \widetilde{\Omega^{(-)}} \langle \Psi_F(A-1, a') | j^\nu(q) | \Psi_I(A) \rangle \langle \Psi_I(A) | j^{\mu\dagger}(q) | \Psi_F(A-1, a') \rangle \Omega^{(-)} \quad (12)$$

$$= s(a) \widetilde{\Omega^{(-)}} j^\nu(q) \sum_{j_z} |\psi_{a'}\rangle \langle \psi_{a'} | j^{\mu\dagger}(q) \Omega^{(-)}. \quad (13)$$

Here, $\psi_{a'}$ is the single-particle wave function of the proton in the a -th shell, $s(a)$ is its spectroscopic factor, and $\widetilde{\Omega^{(-)}}$ is the transpose of $\Omega^{(-)}$.

As we define $\hat{\mathbf{s}}$ in the rest frame of the proton, we decompose the trace in Eq. (8) in terms of the spin-polarization response functions using the (right-handed) coordinate system in that frame. We write the basis vectors of the coordinate system as $(\hat{\mathbf{n}}, \hat{\mathbf{l}}, \hat{\mathbf{t}})$. The spin-polarization is projected onto these vectors as $\mathcal{S}_n = \hat{\mathbf{n}} \cdot \hat{\mathbf{s}}$, $\mathcal{S}_l = \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}$, and $\mathcal{S}_t = \hat{\mathbf{t}} \cdot \hat{\mathbf{s}}$. When the trace in Eq. (8) is expressed in terms of these spin projections, the spin-polarization response

functions, various R^n , R^l , and R^t , emerge in the coefficients of the spin projections, as seen below.

The differential cross section of the $(\vec{e}, e'\vec{p})$ reaction ejecting a proton with h and $\hat{\mathbf{s}}$ is now written in its full form,

$$\begin{aligned} \left(\frac{d\sigma}{dE_{\mathbf{k}'} d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}'}} \right)_{h, \hat{\mathbf{s}}} &= \frac{1}{2} \left(\frac{d\sigma}{dE_{\mathbf{k}'} d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}'}} \right)_h + \left[\left(\frac{d\sigma}{dE_{\mathbf{k}'} d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}'}} \right)_{h, \hat{\mathbf{s}}} - \frac{1}{2} \left(\frac{d\sigma}{dE_{\mathbf{k}'} d\Omega_{\mathbf{k}'} d\Omega_{\mathbf{p}'}} \right)_h \right] \\ &\equiv \frac{1}{2} \sigma(h, 0) + \sigma(h, \hat{\mathbf{s}}), \end{aligned} \quad (14)$$

where $\sigma(h, 0)$ is the differential cross section for $(\vec{e}, e'p)$ and is given by

$$\begin{aligned} \sigma(h, 0) &= \frac{M|\mathbf{p}'|}{(2\pi)^3} \left(\frac{d\sigma}{d\Omega_{\mathbf{k}'}} \right)_{Mott} \\ &\cdot \{v_L R_L + v_T R_T + v_{TT} R_{TT} \cos 2\beta + v_{LT} R_{LT} \sin \beta + h v_{LT'} R_{LT'} \cos \beta\}. \end{aligned} \quad (15)$$

$\sigma(h, \hat{\mathbf{s}})$ is the polarized part of the $(\vec{e}, e'\vec{p})$ differential cross section and is given by

$$\begin{aligned} \sigma(h, \hat{\mathbf{s}}) &= \frac{M|\mathbf{p}'|}{2(2\pi)^3} \left(\frac{d\sigma}{d\Omega_{\mathbf{k}'}} \right)_{Mott} \\ &\cdot \{[v_L R_L^n + v_T R_T^n + v_{TT} R_{TT}^n \cos 2\beta + v_{LT} R_{LT}^n \sin \beta + h v_{LT'} R_{LT'}^n \cos \beta] \mathcal{S}_n \\ &\quad + [v_{TT} R_{TT}^l \sin 2\beta + v_{LT} R_{LT}^n \cos \beta + h(v_{LT'} R_{LT'}^n \sin \beta + v_{TT'} R_{TT'}^n)] \mathcal{S}_l \\ &\quad + [v_{TT} R_{TT}^t \sin 2\beta + v_{LT} R_{LT}^t \cos \beta + h(v_{LT'} R_{LT'}^t \sin \beta + v_{TT'} R_{TT'}^t)] \mathcal{S}_t\} \\ &\equiv N_n \mathcal{S}_n + N_l \mathcal{S}_l + N_t \mathcal{S}_t, \end{aligned} \quad (16)$$

where β is the azimuthal angle of \mathbf{p}' as illustrated in Fig. 1; and v 's ($v_L, v_T, v_{TT}, v_{LT}, v_{LT'}$, and $v_{TT'}$) are kinematic factors, depending only on θ, \mathbf{q}^2 , and q^2 . For completeness, in the Appendix we list the relations between the response functions and the nuclear tensor, and the explicit forms of the kinematic factors.

In the experiments planned at the TJNAF, simplified kinematics is applied to reduce the number of the response functions involved: The in-plane kinematics of $\beta = n\pi$ is used for polarized beams¹² and for unpolarized ($h = 0$) beams¹³. In the latter case, the induced polarization yields the helicity-independent (nonzero) normal polarization component. The differential cross section for this $(e, e'\vec{p})$ is written in terms of the preceding $\sigma(h, 0)$ and N_n (but setting $\beta = n\pi$) as

$$\left(\frac{d\sigma}{dE_{\mathbf{k}'}d\Omega_{\mathbf{k}'}d\Omega_{\mathbf{p}'}}\right)_{h,\hat{\mathbf{s}}} = \frac{1}{2}\sigma(h,0)_{\beta=n\pi}[1+P_n], \quad (17)$$

where

$$P_n = [N_n/\sigma(h,0)]_{\beta=n\pi}. \quad (18)$$

In Section II D, we discuss our numerical results of P_n .

In this work, we use the one-body current operator in free space,

$$j^\mu(q) = \gamma^0 \left[F_1(q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2(q^2)\sigma^{\mu\nu}q_\nu \right], \quad (19)$$

by neglecting off-shell effects involved in the current²⁸. Different prescriptions for the off-shell extension of the current, as well as for recovering the current conservation, are recently discussed²⁹ and will be commented on in Section IV. In this work, we use the standard dipole function for the Dirac and the Pauli form factors $F_1(q^2)$ and $F_2(q^2)$ (with $\kappa = 1.79$), except when noted otherwise.

B. Dirac Eikonal Approximation

The initial- and final-state proton wave functions, $\psi_{a'}(\mathbf{r})$ and $\psi_{\mathbf{p}',s}^{(-)}(\mathbf{r})$ satisfy the Dirac equation with the scalar potential V_s , and the vector potential V_v . $\psi_{a'}(\mathbf{r})$ is the quantum-hydrodynamical wave function in the Hartree approximation²³, and is expressed in the standard form²⁷,

$$\psi_{a'}(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} iG_{n,\kappa}(r)\Phi_{\kappa,j_z}(\Omega) \\ -F_{n,\kappa}(r)\Phi_{-\kappa,j_z}(\Omega) \end{pmatrix} \quad (20)$$

for the nuclear shell state a with $a' = (n, j, l, j_z)$, where j and l are specified through a quantum number κ . The wave function is normalized to unity, and $\Phi_{\pm\kappa,j_z}$ are the spin spherical harmonics for the solid angle, Ω .

The continuum-state wave function of the proton with the momentum \mathbf{p}' and the spin s is expressed as

$$\psi_{\mathbf{p}',s} = \begin{pmatrix} u_{\mathbf{p}',s} \\ w_{\mathbf{p}',s} \end{pmatrix}, \quad (21)$$

where each component satisfies

$$\begin{aligned} \left[\frac{-\nabla^2}{2M} + V_C + V_{SO}(\sigma \cdot \mathbf{L} - i\mathbf{r} \cdot \mathbf{p}') \right] u_{\mathbf{p}',s} &= \frac{\mathbf{p}'^2}{2M} u_{\mathbf{p}',s} \\ w_{\mathbf{p}',s} &= -\frac{i}{D(r)}(\sigma \cdot \nabla) u_{\mathbf{p}',s}, \end{aligned} \quad (22)$$

where $D(r) = E + M + V_s(r) - V_v(r)$. Here, V_C and V_{SO} are the central and spin-orbit potentials related to V_s and V_v by

$$\begin{aligned} V_C(r) &= V_s + \frac{E}{M} V_v + \frac{V_s^2 - V_v^2}{2M} \\ V_{SO}(r) &= \frac{1}{2MD(r)} \frac{1}{r} \frac{d}{dr} [V_v - V_s]. \end{aligned} \quad (23)$$

The solution of Eq. (22) with the incoming boundary condition is given, in the eikonal approximation, by

$$\psi_{\mathbf{p}',s}^{(-)}(\mathbf{r}) = \left(\frac{E_{\mathbf{p}'} + M}{2E_{\mathbf{p}'}} \right)^{1/2} \begin{pmatrix} 1 \\ -iD(r)^{-1}(\sigma \cdot \nabla) \end{pmatrix} e^{i\mathbf{p}' \cdot \mathbf{r}} e^{iS(\mathbf{r})} \chi_s. \quad (24)$$

Here, $S(\mathbf{r})$ is the eikonal phase,

$$S(\mathbf{r}) = \frac{M}{p'} \int_z^\infty dz' \{ V_C(z', \mathbf{b}) + V_{SO}(z', \mathbf{b}) [\sigma \cdot \mathbf{b} \times \mathbf{p}' - ip'z'] \}, \quad (25)$$

where $\mathbf{r} = z \mathbf{e}_z + \mathbf{b} \mathbf{e}_\perp$ with \mathbf{e}_z and \mathbf{e}_\perp being the longitudinal and transverse unit vectors along the direction of \mathbf{p}' . In this work, we are interested in each contribution of the central and spin-orbit forces to the 18 spin-dependent response functions. We implement this by switching on and off V_C and V_{SO} in Eq. (22).

III. NUMERICAL RESULTS

We now describe the numerical results of the spin-dependent response functions for the $(\vec{e}', e' \vec{p})$ reaction, taking ^{16}O as an example. After establishing the accuracy of the eikonal

approximation (in Subsection A), we illustrate the response functions and examine effects of the spin-orbital force (in Subsection B) and of the nucleon electromagnetic form factors (in Subsection C). We also show the normal-component polarization relevant to an experiment planned at TJNAF¹³ (in Subsection D). We present the results at two kinetic energies of the knock-out proton, $T_{p'} = 0.515$ GeV and 3.179 GeV. These energies correspond to the extreme energies in the experiment¹³. At the lower energy of 0.515 GeV, we compare the response functions calculated by the eikonal and partial-wave decomposition methods. Since no detailed phenomenological optical potential is available at these energies, we use the optical potential in the lowest-order impulse approximation, the so-called $f\rho$ -form, where the nuclear density ρ is taken from the Hartree mean-field nuclear wave function. The description of this method in Dirac formalism is elaborated in Ref.³⁰ and is summarized in¹⁹. We use the pN -scattering amplitudes from the phase-shift analyses of Ref.³¹ and Ref.³² for $T_{p'} = 0.515$ and 3.179 GeV, respectively.

A. Dirac Eikonal Approximation vs. Partial-Wave Decomposition Method

To compare the Dirac eikonal and partial-wave decomposition methods, we select ten representative response functions out of the full 18 functions, and show the results at $T_{p'} = 0.515$ GeV ($|\mathbf{p}| = 1.113\text{GeV}/c$) with $Q^2 \equiv -q^2 = 1(\text{GeV}/c)^2$ in Fig. 2. The response functions are shown in the commonly used kinematics in the low energies, as a function of the magnitude of the recoil momentum of the residual nucleus, $|\mathbf{p}' - \mathbf{q}|$, at a constant momentum transfer $|\mathbf{q}|$ (here, $|\mathbf{q}| = 1.113\text{GeV}/c$).

The Dirac partial-wave decomposition method is fully described in Ref.¹⁴, and the response functions by the method shown in Fig. 2 are provided to us by J. W. Van Orden³⁴. The response functions in Fig. 2 by the two methods are obtained in the same kinematics, using the same input parameters together with the Höhler nucleon electromagnetic form factor³³. Figure 2 shows that the results by the two methods are quite close, within 10% at the peak for all response functions shown. The exception is with R_{TT}^n , for which the

discrepancy at the peak is larger (about 20%). Note that a similar, relatively large ($\sim 20\%$) discrepancy is seen with one of the t -component response function, R_{TT}^t (not shown here).

In order to solidify this comparison, we repeat the comparison at $T_{p'} = 135\text{MeV}$ and find the discrepancy to be much larger, typically of 30-40%, and even larger (80 – 100%) for the transverse responses (R_{TT}^t , R_{TT}^n and R_{TT}^l). (We do not exhibit the 135MeV results in order to keep the number of figures reasonable.) As we go up to the GeV region, the number of the partial waves naturally increases, and the partial-wave decomposition method becomes more elaborate and eventually become impractical. On the other hand, the eikonal becomes more accurate as the ratio of $T_{p'}$ and (the magnitude of) the pN potential increases. Though we have no partial-wave decomposition result available to compare at the GeV region, we expect the eikonal method to be reasonably accurate. The Dirac eikonal method should be the practical, reasonably reliable method for examining the final-state interaction in the high-energy ($\vec{e}, e'\vec{p}$) reaction.

B. Spin-Orbit Force

Figures 3 and 4 show the complete set of 18 spin-dependent response functions for the proton knock-out from the $p_{1/2}$ -shell with the kinetic energy of $T_{p'} = 0.515\text{ GeV}$ (the same kinematics as that used in Subsection II B, $|\mathbf{p}'| = |\mathbf{q}| = 1.133\text{ GeV}/c$). The response functions are calculated with and without the final-state interaction (that is, DWIA and PWIA, respectively.) The DWIA responses are generally smaller in magnitude than the PWIA responses, as a consequence of the absorption in the final-state interaction. The largest response function is R_T among the unpolarized response functions, R_L, R_T, R_{TT}, R_{LT} and $R_{LT'}$, and dominates the unpolarized cross section.

The helicity-dependent response function, $R_{LT'}$, vanishes in the absence of the final-state interaction and is a quantity useful for the investigation of the proton-flux attenuation by the final-state interaction. At the parallel kinematics (i.e., $|\mathbf{p}' - \mathbf{q}| = 0$), R_{TT} , R_{LT} and $R_{LT'}$ vanish. At $T_{p'} = 0.135\text{ GeV}$, it was observed¹⁴ by the partial-wave decomposition

calculation, that the sign of R_{TT} changes by the inclusion of the final-state interaction for the proton knocked out from the $1p_{1/2}$ -shell. We find the same to occur at this energy and also at $T_{p'} = 3.197$ GeV. Figure 5 shows the response functions R 's and R^n 's for the proton knocked out from the $1p_{3/2}$ -shell. Here, the sign of R_{TT} remains the same with the inclusion of the final-state interaction as is the case at $T_{p'} = 0.135$ GeV¹⁴. The response functions for the polarized proton in the **n**, **l** and **t** directions are also shown in Fig.4, many of which vanish in the absence of the final-state interaction.

Figures 6 and 7 illustrate the response functions for the proton knocked out from the $1p_{1/2}$ -shell at $T_{p'} = 3.179$ GeV ($|\mathbf{p}'| = 4.024$ GeV/c) with $|\mathbf{q}| = 4.024$ GeV/c, and $Q^2 = 6$ (GeV/c)². The magnitude of the response functions at this energy is typically smaller by two orders of magnitude than those at $T_{p'} = 0.515$ GeV. This reduction is caused mostly by the Q^2 dependence of the nucleon electromagnetic form factor, the square of which contributes to the response functions. Clearly, a further increase in Q^2 that is expected in future experiments will reduce considerably the magnitude of the response functions.

Note that in order to keep the number of figures reasonable, we have selected the figures to be presented in this work: We show the full set of the response functions for the proton knocked out from the $1p_{1/2}$ -shell at $T_{p'} = 0.515$ and 3.179 GeV, so that one could compare them with the lower-energy result at $T_{p'} = 0.135$ GeV in Ref.¹⁶. We also show the unpolarized and normal-component, polarized response functions (R 's and R^n 's, respectively) for the case of the $1p_{3/2}$ -shell because of their greater contributions to P_n and the spin-orbital effects than the R^l 's and R^t 's.

It is interesting to examine how the spin-dependent force in the final-state interaction affects the response functions. For this purpose, we repeated the calculation by omitting the spin-orbit force from the final-state interaction ($V_{SO} = 0$). The resultant response functions are shown in dash lines in Figs. 3 - 9. We see that the interesting sign change of R_{TT} noted above can be attributed to the effect of the spin-orbit force, as clearly demonstrated in R_{TT} of Fig.3. As a consequence of the interference between the effects of the central and spin-orbital interactions, the spin-orbital force *increases* the TT component of the outgoing flux

of the proton. By comparing R_{TT} of Fig. 3 and Fig. 6, we observe that this effect becomes relatively weaker as the energy increases.

The response function for the normally polarized response state R_T^n has a similar feature, but here, the plane-wave response and the response without the spin-orbital force vanish. That is, the central force does not affect R_T^n , but only the spin-orbital force does. The situation is opposite in R_{TT}^n , in which the effect of the final-state interaction is dominated by the central force. The same features as described here are also seen in Fig. 5 in the case of the $1p_{3/2}$ -shell.

Finally, we note that the signs of response functions are generally opposite for the $1p_{1/2}$ - and $1p_{3/2}$ -shell, except for the cases of R_L, R_T, R_{LT} , and $R_{LT'}^n$.

C. Electromagnetic Form Factors of The Nucleon

We also examine the dependence of response functions on the structure of the nucleon electromagnetic current. Figure 8 illustrates the response functions with the Dirac-type current (γ^μ) only (obtained by setting $F_2(q^2) = 0$ and $F_1(q^2) \neq 0$), in the case of the proton knock-out from the $1p_{1/2}$ -shell at $T_{p'} = 0.515$ GeV. The response functions with the Pauli current ($\sigma^{\mu\nu} q_\nu$) only (obtained by setting $F_1(q^2) = 0$ and $F_2(q^2) \neq 0$) are shown in Fig. 9. Note that the response functions shown in Fig. 3 correspond (roughly speaking) to the sum of these two (F_1 and F_2), including the interference between them. We observe that these two types of the electromagnetic current are equally important for most of the response functions. Figures 8 and 9 also include similar calculations without the spin-orbital force in the final-state interaction. We also observe the same feature in this case.

In the cases of the helicity-dependent response functions, $R_{LT'}$, $R_{LT'}^n$, and R_{TT} , the contributions of the Dirac-type and the Pauli-type currents have opposite signs, while the signs remain the same in the other response functions. The neutron has a net zero charge, and its Dirac form factor is extremely small ($F_1 \simeq 0$), as is well-known from the fact that the Sachs charge radius of the neutron is almost completely saturated by the magnetic radius.

The response functions shown in Fig. 9 are thus expected to be similar (sign-wise and magnitude-wise) to the response functions for the $(\vec{e}, e'\vec{n})$ reaction. We have confirmed this expectation by calculating the response functions for the $(\vec{e}, e'\vec{n})$ reaction with the realistic neutron form factors. Note that we are neglecting the charge-exchange contribution to the $(\vec{e}, e'\vec{n})$ reaction, but the contribution is expected to be relatively small in the GeV energy region. A further note on a more detailed feature: The helicity-dependent response functions, $R_{LT'}$ and $R_{LT'}^n$, have opposite signs in $(\vec{e}, e'\vec{n})$ and $(\vec{e}, e'\vec{p})$.

D. Polarization of the Knocked-out Nucleon: P_n

The normal-component polarization of the outgoing proton, P_n , can be observed in the $(e, e'\vec{p})$ reaction with an unpolarized electron beam¹³. P_n is expressed in terms of the response functions as shown in Eqs. (15)-(18). Figure 10 illustrates P_n for the proton knock-out from the the $1p_{1/2}$ - and $1p_{3/2}$ -shells at $T_{p'} = 0.515$ GeV. In the absence of the final-state interaction, the normal component of the spin-dependent response functions R_L^n, R_T^n, R_{TT}^n and R_{LT}^n vanish, so that $P_n = 0$ in the PWIA. P_n for the $1p_{1/2}$ shell is negative for $|\mathbf{p}' - \mathbf{q}| < 1.5 \text{ fm}^{-1}$, while P_n for the $1p_{3/2}$ -shell is positive for $|\mathbf{p}' - \mathbf{q}| < 1 \text{ fm}^{-1}$. The polarization induced only by the central force V_C is also shown in Fig. 10. Similar results for $T_{p'} = 3.179$ GeV are shown in Fig.11. The nuclear-recoil dependence of P_n is similar at both energies, but its magnitude is considerably smaller (by more than 40%) at $T_{p'} = 3.179$ GeV than at $T_{p'} = 0.515$ GeV, even becoming comparable to the expected experimental accuracy $\Delta P_n \simeq 0.5$ ¹³.

The polarization of the outgoing proton P_n is induced by the final-state interaction, so it vanishes in the absence of the final-state interaction. In fact, P_n is insensitive to the structure of the electromagnetic current: Numerically we find P_n for the two cases, $F_1(q^2) \neq 0$ with $F_2(q^2) = 0$ and $F_2(q^2) \neq 0$ with $F_1(q^2) = 0$, to be practically identical.

We have also examined P_n for the $(\vec{e}, e'\vec{n})$ and $(\vec{e}, e'\vec{p})$ reactions at different $T_{p'}$ from different shell-orbits. P_n for the two reactions are found to be almost identical, but as noted

previously, our calculation does not include the charge-exchange interaction.

IV. DISCUSSION

We comment on the two important effects that we have neglected in this work.

The current conservation. A DWIA calculation of the $(\vec{e}, e'\vec{p})$ amplitude suffers from the violation of current conservation. The violation arises basically in the truncation of the many-body degrees of freedom by reduction to the one nucleon problem of the mean-field theory. It is also closely related to the treatment of the off-shell effects.

The current conservation implies a constraint on the nuclear matrix elements of the longitudinal and time components, $q^0 J_{\alpha, \hat{s}}^0(q) = |\mathbf{q}| J_{\alpha, \hat{s}}^L(q)$. A quantity such as $(R_L - \widetilde{R}_L)/(R_L + \widetilde{R}_L)$ would provide a measure of the violation¹⁴. Here, the longitudinal response function R_L is calculated by the use of $J_{\alpha, \hat{s}}^L(q)$, and the \widetilde{R}_L is by the use of $q^0 J_{\alpha, \hat{s}}^0(q)/|\mathbf{q}|$. Though the quantity was found to reach nearly 40 % at $T_{p'} = 135$ MeV¹⁴, it has been estimated to be much less, $\leq 10\%$, for $T_{p'} > 0.515$ GeV¹⁹. The latter high-energy estimate is comparable to other uncertainties in our calculation, such as those in the optical-potential parameters. Note, however, the normal-component polarizations, which are important to P_n , for example, would be less affected by the nonconservation, because these quantities depend mostly on the transverse components that are not associated with the current conservation.

The off-shell effects: The issue of the nonconserved current is complicated because of the off-shell effects because there is no unique way to recover the current conservation for the off-shell nucleon. For example, other forms of the one-body current operator $j^\mu(q)$ that are equivalent to Eq. (18) by means of the Gordon decomposition are no longer equivalent²⁸. Recently, in PWBA, the off-shell effects for $(e, e'p)$ are estimated to be $\leq 10\%$ in the GeV region after the current conservation is imposed in various ways²⁹.

From these, we suspect that the important physics neglected in this work could contribute appreciably. Clearly, more refined work is needed to establish reliable results.

V. SUMMARY AND CONCLUSION

In this work, we have presented the first DWIA calculation of the spin-polarization response functions of the $(\vec{e}, e' \vec{p})$ reaction in the GeV region. As such, we neglect some important physics such as the nuclear current conservation and the off-shell effects. The Dirac eikonal formalism that we used seems to agree well with the partial-wave expansion method in the GeV region.

Our findings are summarized as follows:

(1) The effect of the final-state interaction in R_{TT} is caused mostly by the spin-orbit interaction, while that in R_T^n is caused *solely* by the spin-orbit force. The effect in R_{TT}^n is caused mostly by the central force. These are the cases for both of the $1p_{1/2}$ -shell and the $1p_{3/2}$ -shell knock-out processes also at both of $T_{p'} = 0.515$ GeV and 3.179 GeV.

(2) Except for the helicity-dependent $R_{LT'}^n$, all normal-component responses have different signs for the $1p_{1/2}$ - and the $1p_{3/2}$ -shell knock-outs. P_n thus receives different contributions from the two different shell-orbits.

(3) The response functions become smaller as Q^2 increases, mostly because of the Q^2 dependence of the electromagnetic form factor of the nucleon.

(4) The contributions of the Dirac and the Pauli currents are equally significant to the response functions, but they contribute with different signs to the helicity-dependent response functions, $R_{LT'}$ and $R_{LT'}^n$.

(5) The nonvanishing value of P_n that is due to the the final-state interactions is insensitive to the structure of the electromagnetic current operator.

Among these, let us make a speculative comment on (1): Because they vanish or almost vanish in the absence of the the spin-orbital, final-state interaction, detailed measurements of R_T^n and R_{TT} may reveal an interesting, spin-dependent process of the small, color-singlet proton that may be produced in the high-energy $(e, e'p)$. So far, no investigation has been made on the spin-dependent process except for a speculative description³⁵. It would be an interesting issue that may reveal more about this strange form of the proton, especially

because most experiments are carried out in the energy region where the process would be incompletely controlled by the perturbative QCD.

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APPENDIX A: KINEMATIC FACTORS OF STRUCTURE FUNCTIONS

The kinematic factors, v 's, in Eqs. (15) and (16) are defined to be $v_L = \frac{Q^4}{\mathbf{q}^4}$, $v_T = \left[\frac{Q^2}{2\mathbf{q}^2} + \tan \theta/2 \right]$, $v_{TT} = \frac{Q^2}{2\mathbf{q}^2}$, $v_{LT} = \frac{Q^2}{\mathbf{q}^2} \left[\frac{Q^2}{\mathbf{q}^2} + \tan^2 \theta/2 \right]^{1/2}$, $v_{LT'} = \frac{Q^2}{\mathbf{q}^2} \tan \theta/2$, and $v_{TT'} = \tan \theta/2 \left[\frac{Q^2}{\mathbf{q}^2} + \tan^2 \theta/2 \right]^{1/2}$ with $Q^2 = -q^2$.

The response functions are obtained by the application of the projection operator $\mathcal{P}_{\mathbf{a}} = |\mathbf{a}\rangle \langle \mathbf{a}| \frac{1}{2} (1 + \sigma \cdot \hat{\mathbf{a}})$ for $\hat{\mathbf{a}} = \hat{\mathbf{n}}, \hat{\mathbf{l}}$ or $\hat{\mathbf{t}}$. More explicitly, they are given by

$$\begin{aligned}
R_L &= Tr\{\widetilde{R}_L \mathbf{I}\}, & R_L^n &= Tr\{\widetilde{R}_L \sigma \cdot \mathbf{n}\}, \\
R_T &= Tr\{\widetilde{R}_T \mathbf{I}\}, & R_T^n &= Tr\{\widetilde{R}_T \sigma \cdot \mathbf{n}\}, \\
R_{TT} &= Tr\{\widetilde{R}_{TT} \mathbf{I}\} / \cos 2\beta, & R_{TT}^n &= Tr\{\widetilde{R}_{TT} \sigma \cdot \mathbf{n}\} / \cos 2\beta, \\
R_{LT} &= Tr\{\widetilde{R}_{LT} \mathbf{I}\} / \sin \beta, & R_{LT}^n &= Tr\{\widetilde{R}_{LT} \sigma \cdot \mathbf{n}\} / \sin \beta, \\
R_{LT'} &= Tr\{\widetilde{R}_{LT'} \mathbf{I}\} / \cos \beta, & R_{LT'}^n &= Tr\{\widetilde{R}_{LT'} \sigma \cdot \mathbf{n}\} / \cos \beta, \\
R_{LT}^t &= Tr\{\widetilde{R}_{LT} \sigma \cdot \mathbf{t}\} / \cos \beta, & R_{LT}^l &= Tr\{\widetilde{R}_{LT} \sigma \cdot \mathbf{l}\} / \cos \beta, \\
R_{TT}^t &= Tr\{\widetilde{R}_{TT} \sigma \cdot \mathbf{t}\} / \sin 2\beta, & R_{TT}^l &= Tr\{\widetilde{R}_{TT} \sigma \cdot \mathbf{l}\} / \sin 2\beta, \\
R_{LT'}^t &= Tr\{\widetilde{R}_{LT'} \sigma \cdot \mathbf{t}\} / \sin \beta, & R_{LT'}^l &= Tr\{\widetilde{R}_{LT'} \sigma \cdot \mathbf{l}\} / \sin \beta, \\
R_{TT'}^t &= Tr\{\widetilde{R}_{TT'} \sigma \cdot \mathbf{t}\}, & R_{TT'}^l &= Tr\{\widetilde{R}_{TT'} \sigma \cdot \mathbf{l}\},
\end{aligned} \tag{A1}$$

where \widetilde{R} 's are given in terms of the nuclear tensor in the Dirac plane-wave, spinor space as

$$\begin{aligned}
\widetilde{R}_L &= \overline{\omega}^{00}, \\
\widetilde{R}_T &= \overline{\omega}^{22} + \overline{\omega}^{11}, \\
\widetilde{R}_{TT} &= \overline{\omega}^{22} - \overline{\omega}^{11}, \\
\widetilde{R}_{LT} &= \overline{\omega}^{20} - \overline{\omega}^{02}, \\
\widetilde{R}_{LT'} &= i(\overline{\omega}^{10} - \overline{\omega}^{01}), \\
\widetilde{R}_{TT'} &= i(\overline{\omega}^{12} - \overline{\omega}^{21}).
\end{aligned} \tag{A2}$$

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FIGURES

FIG. 1. The coordinate system and kinematical variables of the $(\vec{e}, e'\vec{p})$ reaction. The notations are the same as those used in Refs. 15 and 16.

FIG. 2. Response functions for the proton knocked out of the $1p_{1/2}$ -shell of ^{16}O with the kinetic energy of $T_{p'} = 0.515$ GeV. $|\mathbf{p} - \mathbf{q}|$ is the magnitude of the recoil momentum of the residual nucleus. The functions calculated by the use of the Dirac eikonal formalism are shown by solid curves, and those by the partial-wave decomposition method are shown by dotted curves.

FIG. 3. Unpolarized and normal-component, polarization response functions for the proton knocked out of the $1p_{1/2}$ -shell of ^{16}O with the kinetic energy of $T_{p'} = 0.515$ GeV. $|\mathbf{p} - \mathbf{q}|$ is the magnitude of the recoil momentum of the residual nucleus. Solid curves are the DWIA results by use of the Dirac eikonal formalism, and dotted curves are the PWIA results. The DWIA results with no spin-orbit potential ($V_{SO} = 0$) are also shown in dashed curves.

FIG. 4. The same as the caption for Fig. 3, except for the **l**- and **t**-component polarization response functions.

FIG. 5. The same as the caption of Fig.3, except that the proton is knocked out of the $1p_{3/2}$ -shell.

FIG. 6. The same as the caption of Fig.4, except that the proton is knocked out with the kinetic energy of $T_{p'} = 3.179$ GeV.

FIG. 7. The same as the caption of Fig.5, except that the proton is knocked out with the kinetic energy of $T_{p'} = 3.179$ GeV.

FIG. 8. The same as the caption of Fig.4, except that the Dirac-type current ($F_1(q^2)\gamma^\mu \neq 0$ and $F_2(q^2)\gamma^\mu = 0$) is used.

FIG. 9. The same as the caption of Fig.4, except that the Pauli-type current ($F_1(q^2)\gamma^\mu = 0$ and $F_2(q^2)\gamma^\mu \neq 0$) is used.

FIG. 10. The normal-component polarization, P_n , for the proton knocked out of the $1p_{1/2}$ - and the $1p_{3/2}$ -shells with the kinetic energy of $T_{p'} = 0.515$ GeV. The dotted curves are calculated only with the central potential, and the solid curves are with the full (central and spin-orbit) potential.

FIG. 11. The same as the caption of Fig.11, except for the proton kinetic energy of $T_{p'} = 3.179$ GeV.